



# Applying complementary energy methods to estimate ground reaction forces of an exoskeleton

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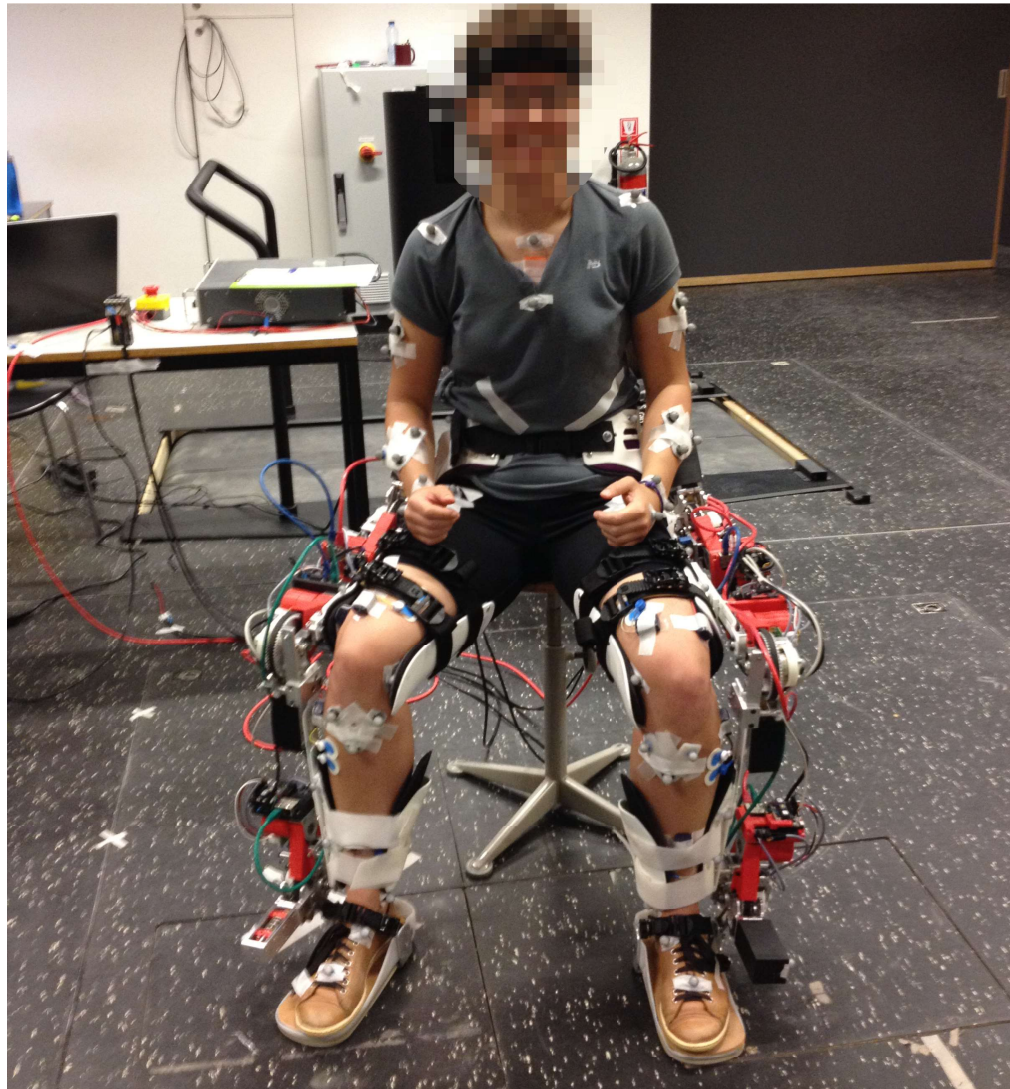
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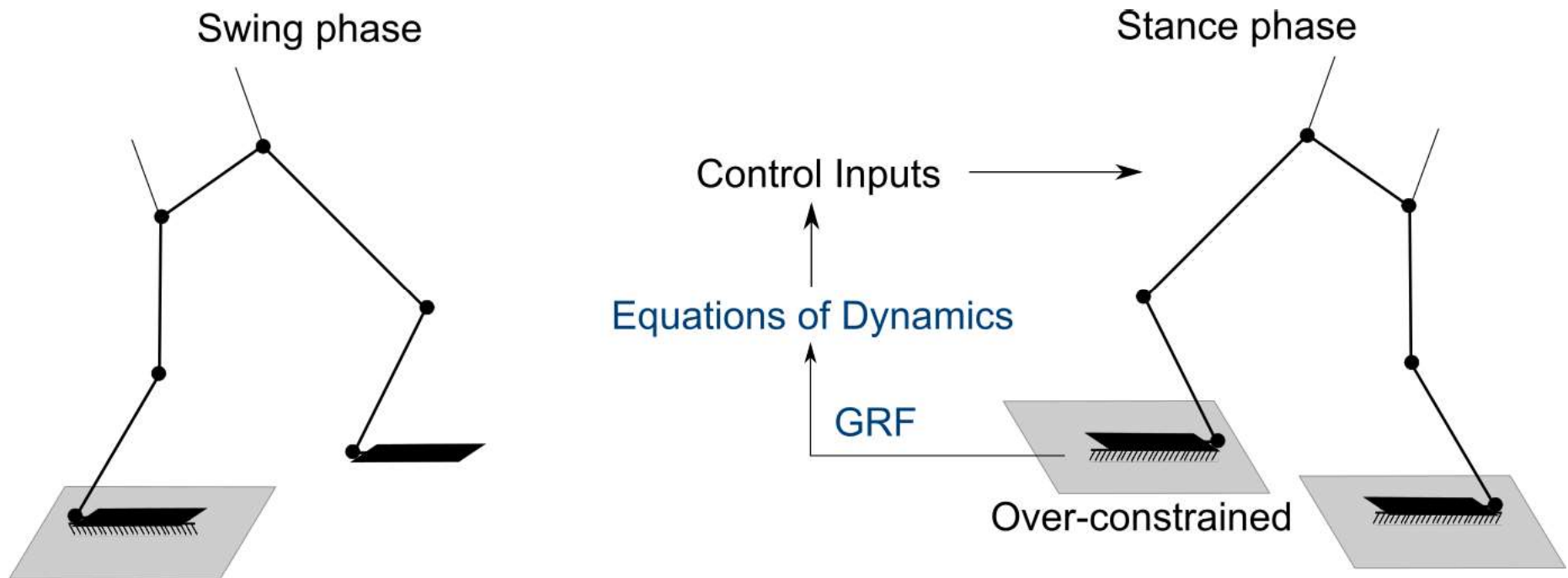
05/02/2018

# Introduction

Lower limb exoskeleton  $\Rightarrow$  mobility assistance/ rehabilitation



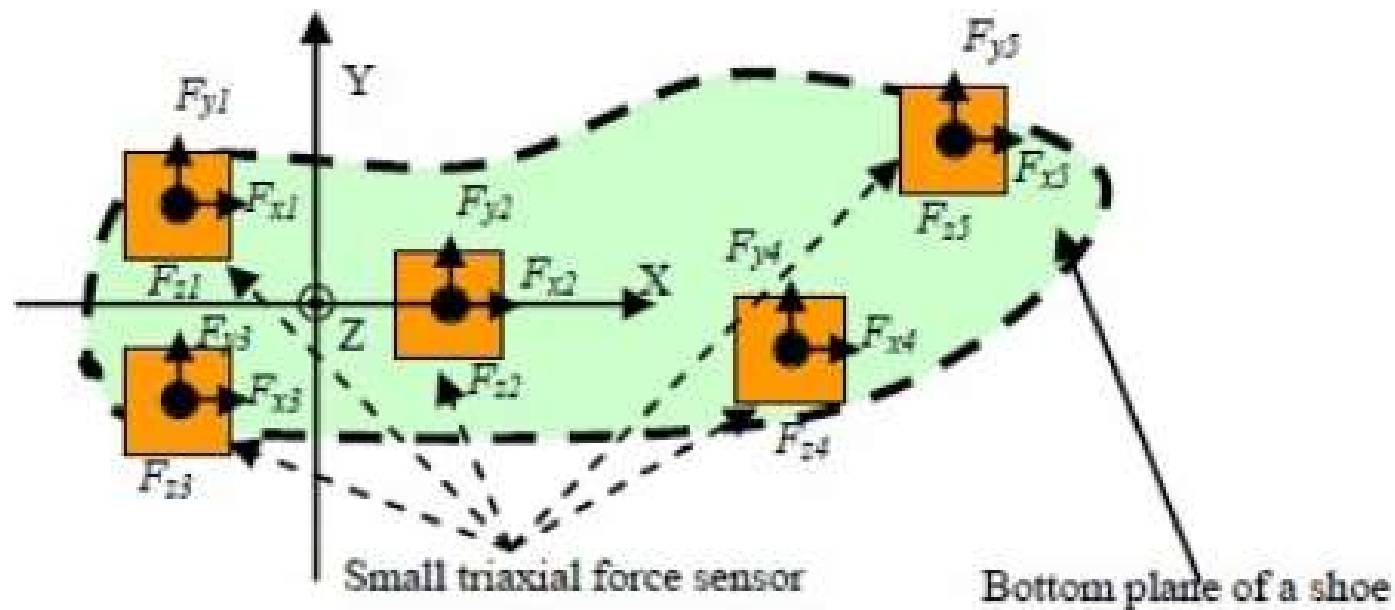
# Introduction



- Method proposed to obtain physically inspired solution
- Applied on stand-alone exoskeleton  $\Rightarrow$  similar to humanoids and other legged robots

# Introduction

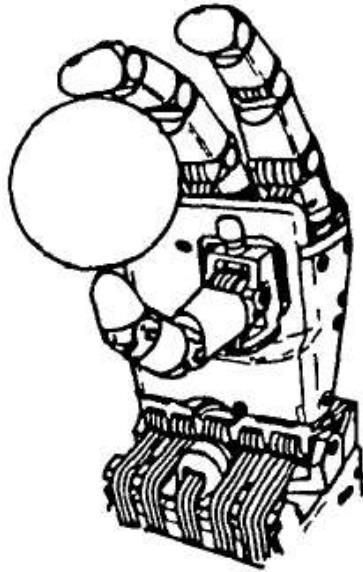
## Research Background



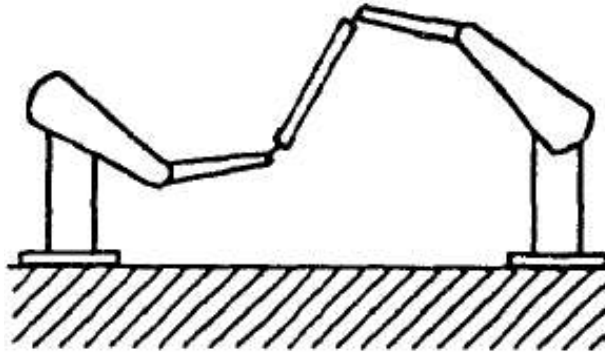
Measuring the GRFs using force cells [Tao et al., 2012]

# Introduction

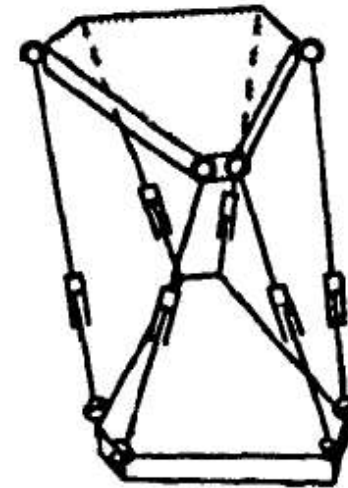
## Research Background



Grasping



Cooperative Robots



Parallel  
Manipulator

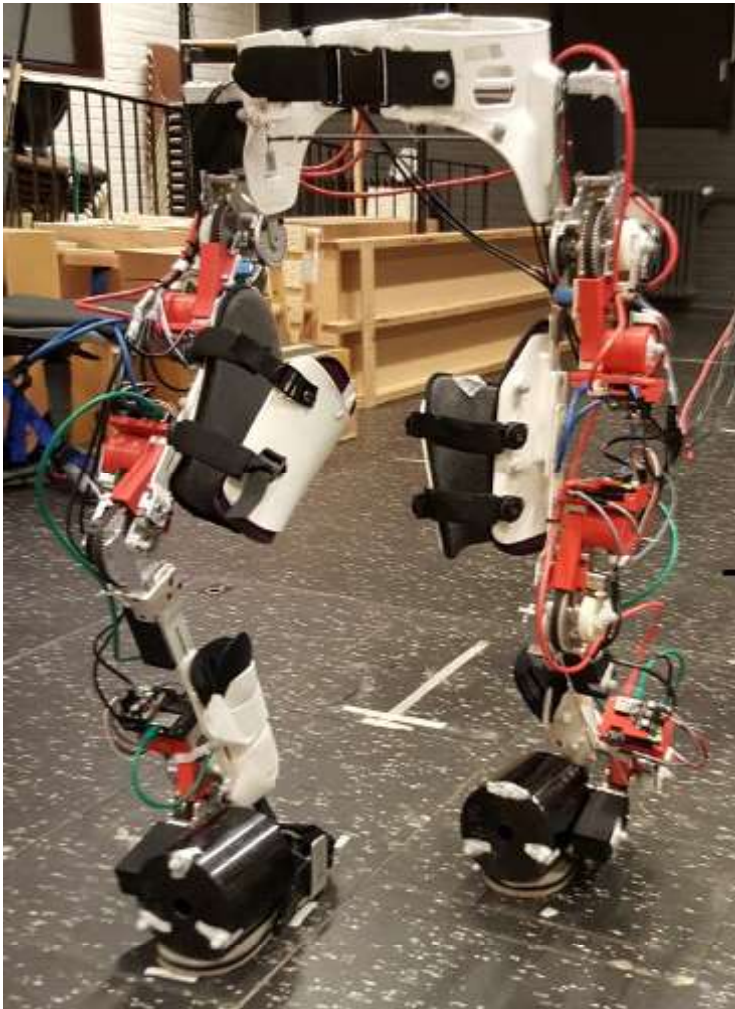
[Nahon, 1993]

Estimating the GRFs using the optimization techniques

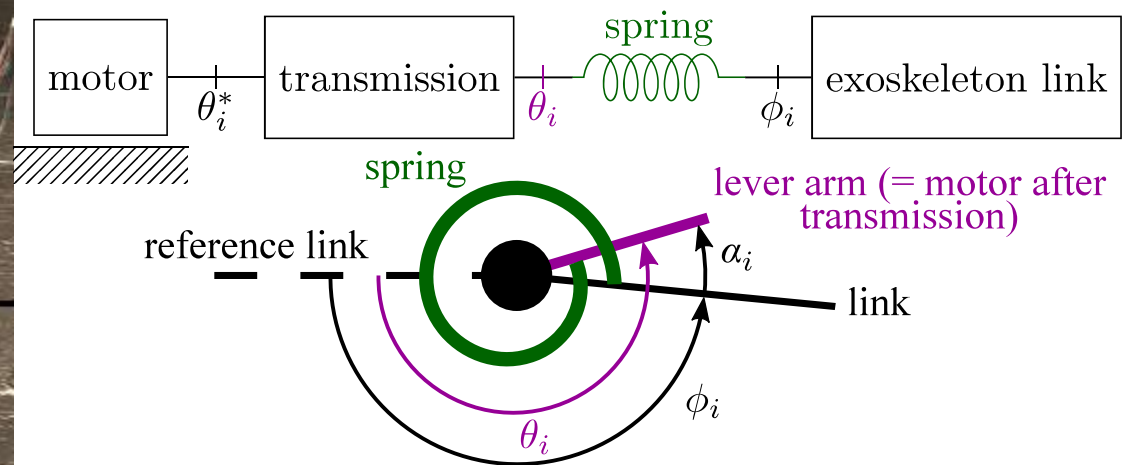


# Exoskeleton

## Design



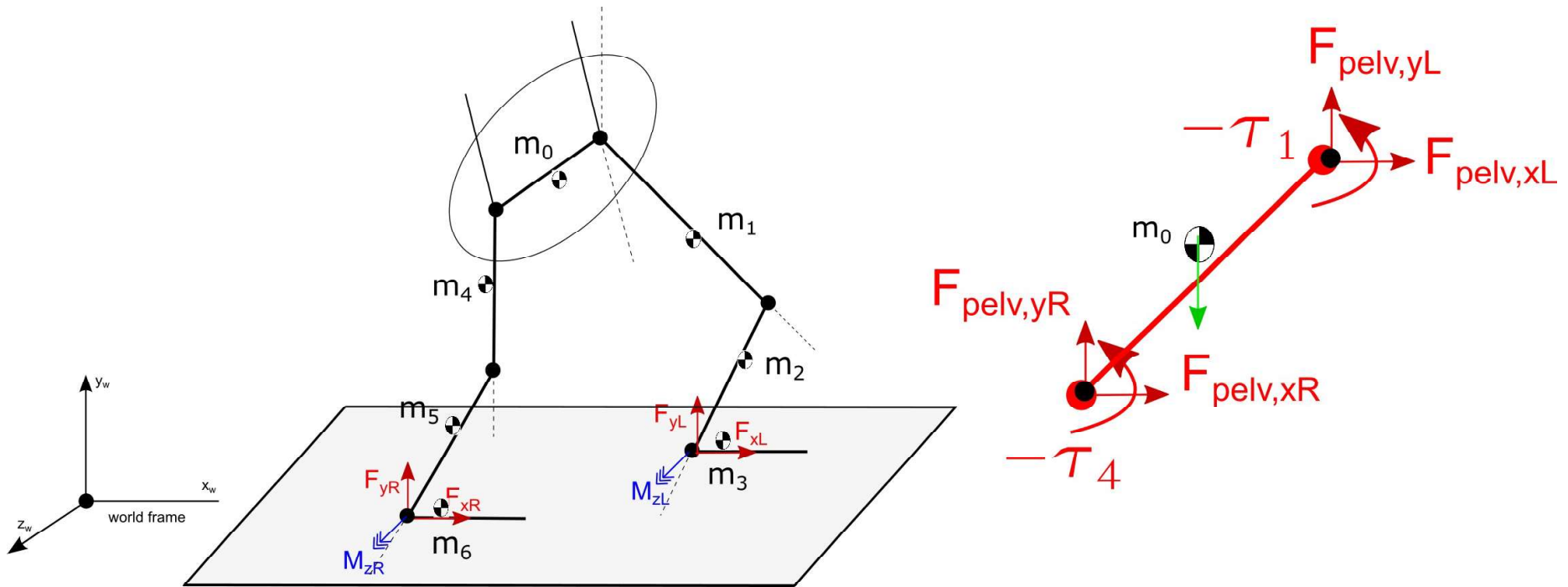
MIRAD Exoskeleton



Serial Elastic Actuator

# Exoskeleton

## Stiffness Model

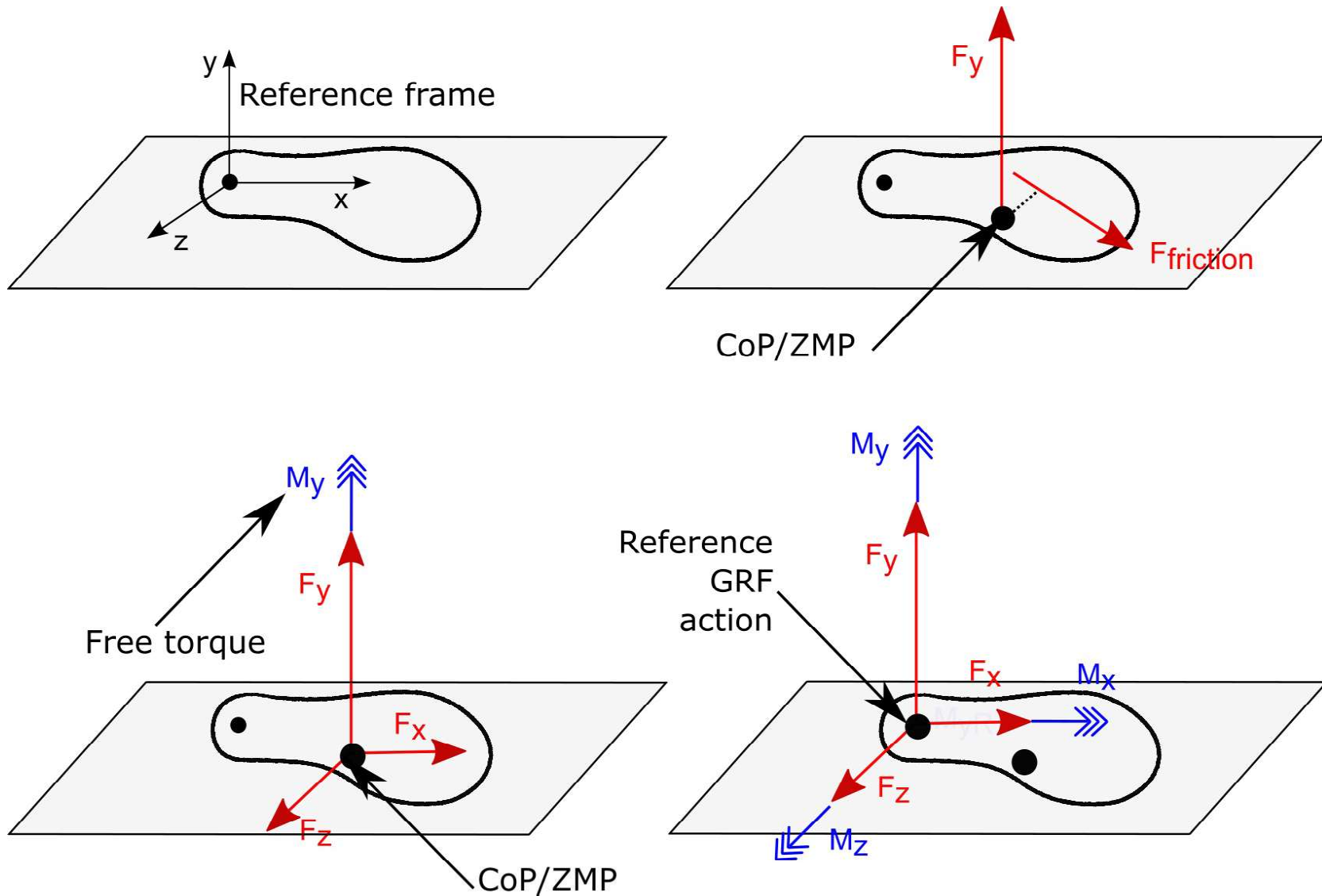


### Compliances in the system

- Joint actuators
- Pelvis joining the two legs
- Links are many times stiffer (neglected)

# Exoskeleton

## GRF model at a foot

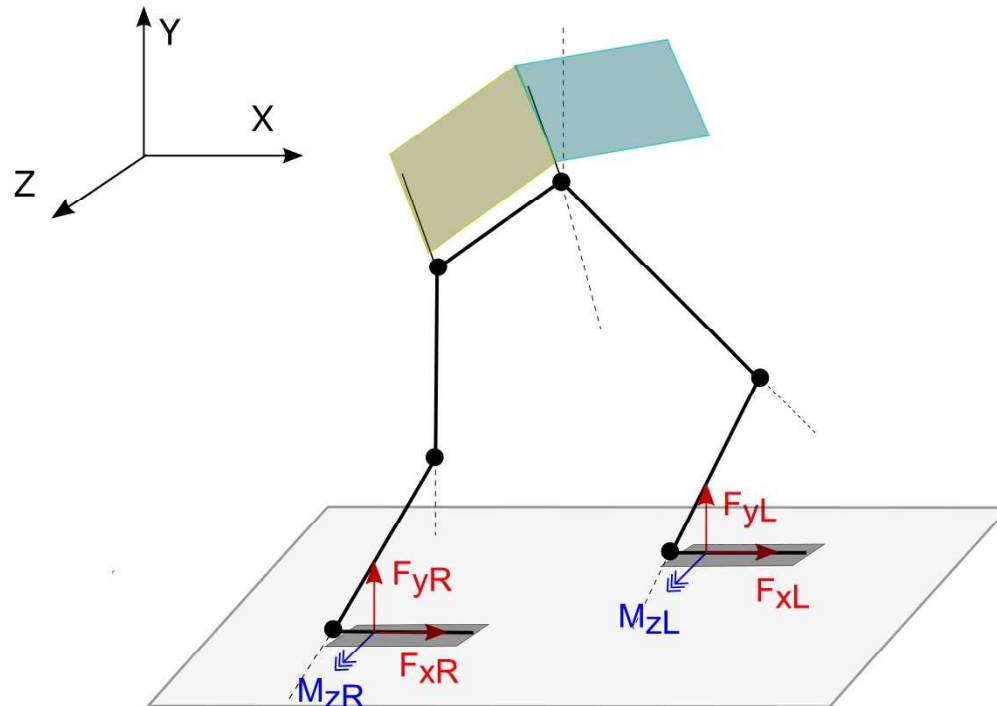




# Exoskeleton

## Dynamic Model

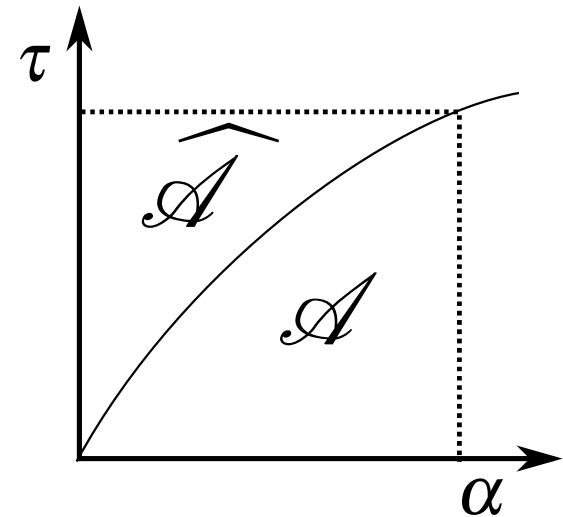
$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = S^T \tau_{spr} + J_c(q)^T w_c$$



# Solution Strategy

## Complementary Energy Methods

- **Equations of compatibility** can be derived from the generalized force approaches which work on a scalar quantity called **complementary energy**



Torque versus deflection curve

- Corollary of the **principle of minimum complementary energy**: Castigliano's theorem, Crotti-Engesser theorem

$$\widehat{\mathcal{A}} = \sum_{i=1}^n \int_0^{\tau_i} \alpha_i(\tau_i) d\tau_i$$
$$\frac{\partial \widehat{\mathcal{A}}}{\partial w_j} = \sum_{i=1}^n \alpha_i(\tau_i) \frac{\partial \tau_i}{\partial w_j} = 0$$

# Solution Strategy

Applied to the exoskeleton

- Equilibrium Equations

$$\mathbf{M}_{flb} \ddot{\mathbf{q}} + \mathbf{C}_{flb} \dot{\mathbf{q}} + \mathbf{g}_{flb} = \mathbf{J}_{Lf,flb}^T \mathbf{w}_L + \mathbf{J}_{Rf,flb}^T \mathbf{w}_R$$

$$\mathbf{M}_L \ddot{\mathbf{q}} + \mathbf{C}_L \dot{\mathbf{q}} + \mathbf{g}_L = \boldsymbol{\tau}_{spr,L} + \mathbf{J}_{Lf,L}^T \mathbf{w}_L$$

$$\mathbf{M}_R \ddot{\mathbf{q}} + \mathbf{C}_R \dot{\mathbf{q}} + \mathbf{g}_R = \boldsymbol{\tau}_{spr,R} + \mathbf{J}_{Rf,R}^T \mathbf{w}_R$$

- Compatibility Equations

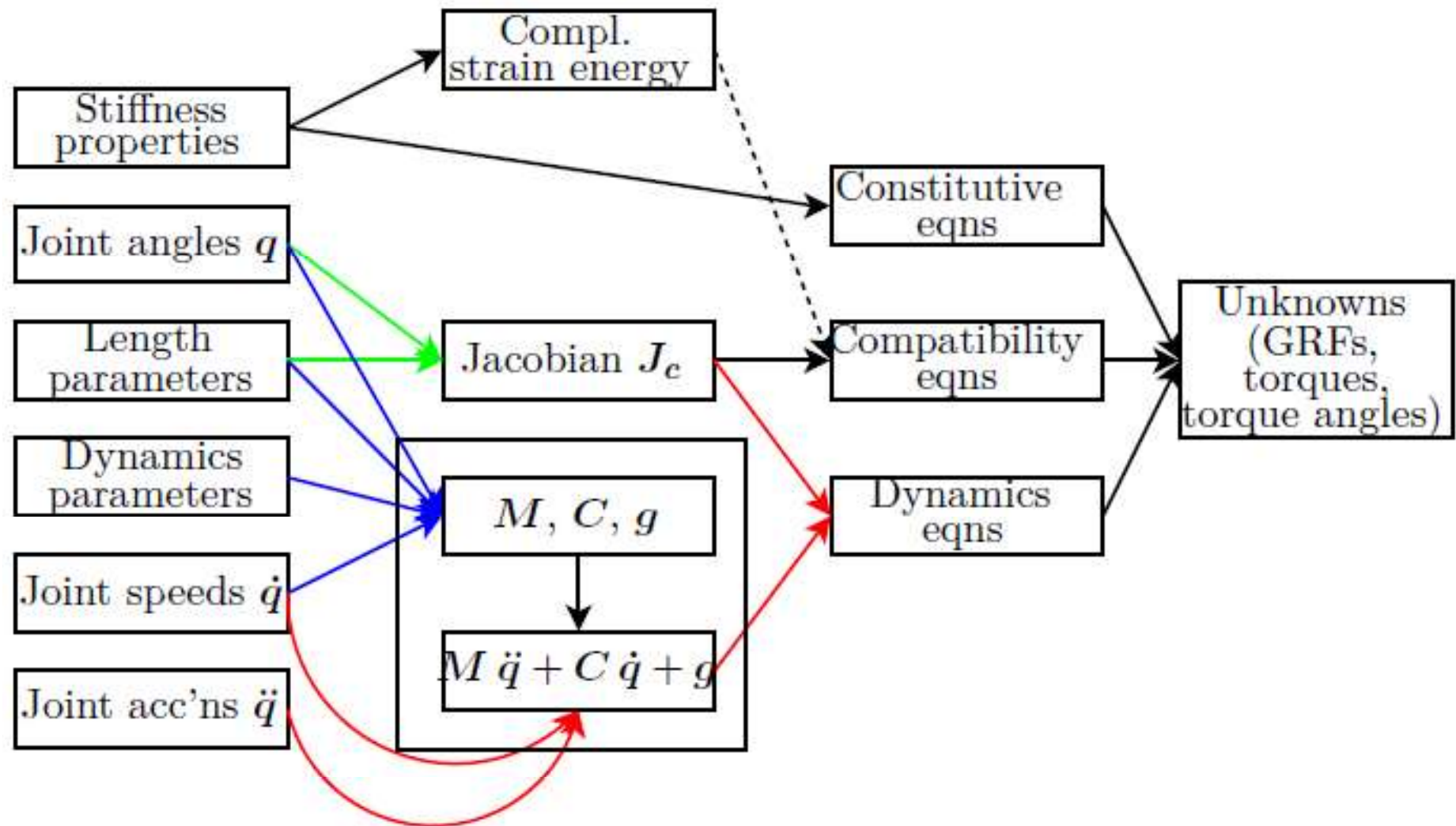
$$((- \mathbf{J}_{Lf,L}^T) (\mathbf{J}_{Lf,flb}^T)^{-1} (- \mathbf{J}_{Rf,flb}^T))^T \boldsymbol{\alpha}_L + (- \mathbf{J}_{Rf,R}^T)^T \boldsymbol{\alpha}_R + [\dots] = \mathbf{0}_{3 \times 1}$$

- Constitutive Laws

$$\tau_i = k_{0,i} + k_{1,i} \alpha_i + k_{2,i} \alpha_i^2 + k_{3,i} \alpha_i^3, \text{ for } i = 1:6$$

# Solution Strategy

## Estimation Algorithm



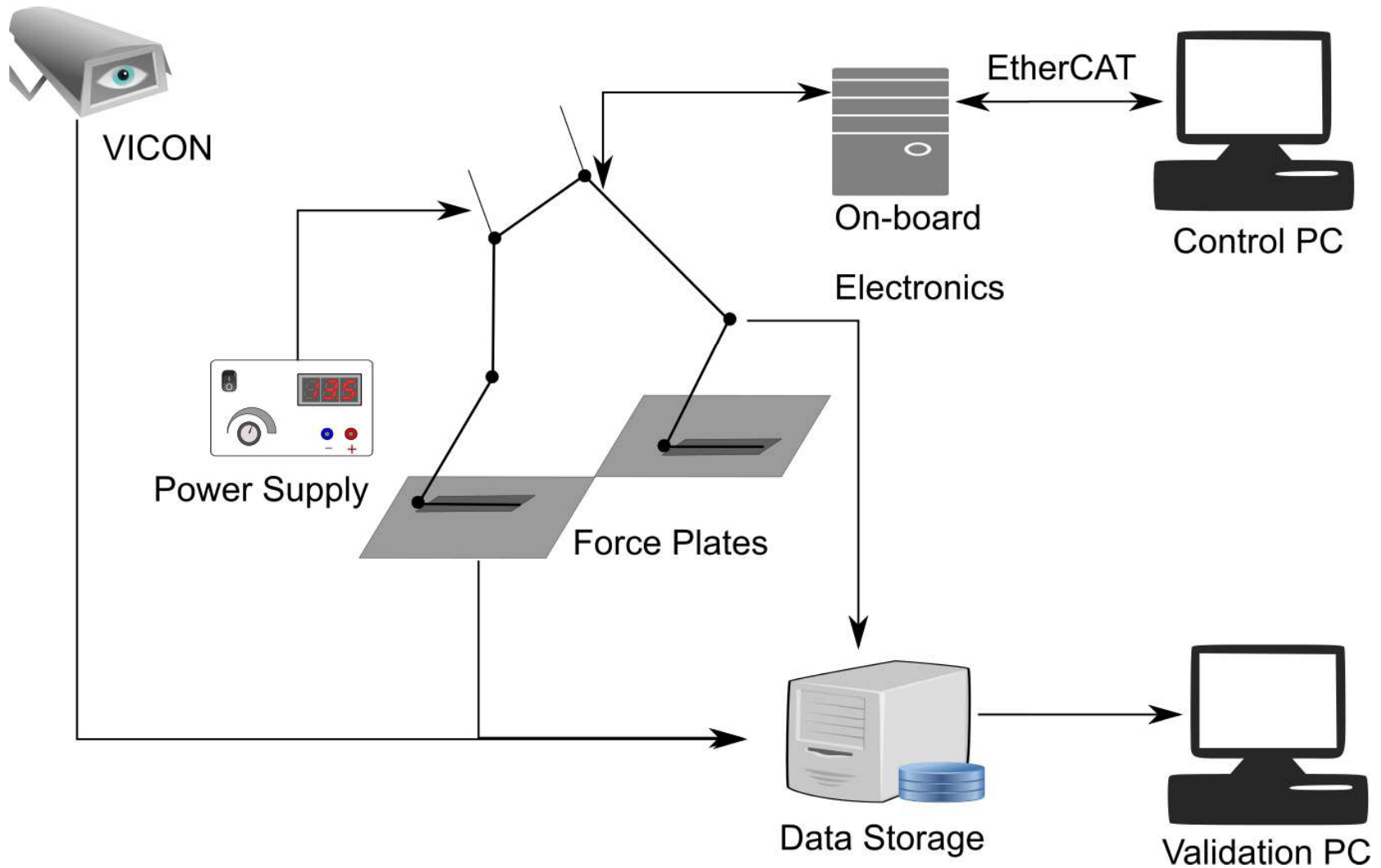
# Solution Strategy

## Optimization Formulation

$$\begin{aligned}
 & \min_{\mathbf{w}_L, \mathbf{w}_R, \boldsymbol{\tau}_{spr,L}, \boldsymbol{\tau}_{spr,R}, \boldsymbol{\alpha}} \sum_{i=1}^6 \left( \int_0^{\tau_i} \alpha_i d\tau_i \right) \\
 & + \frac{1}{2K_{pelv,x}} (2F_{x,R} - \overline{\tau_{dyn,flb,x}})^2 + \frac{1}{2K_{pelv,y}} (2F_{y,R} - \overline{\tau_{dyn,flb,y}})^2 \\
 & \text{subject to} \\
 & \mathbf{M}_{flb} \ddot{\mathbf{q}} + \mathbf{C}_{flb} \dot{\mathbf{q}} + \mathbf{g}_{flb} = \mathbf{J}_{Lf,flb}^T \mathbf{w}_L + \mathbf{J}_{Rf,flb}^T \mathbf{w}_R \\
 & \mathbf{M}_L \ddot{\mathbf{q}} + \mathbf{C}_L \dot{\mathbf{q}} + \mathbf{g}_L = \boldsymbol{\tau}_{spr,L} + \mathbf{J}_{Lf,L}^T \mathbf{w}_L \\
 & \mathbf{M}_R \ddot{\mathbf{q}} + \mathbf{C}_R \dot{\mathbf{q}} + \mathbf{g}_R = \boldsymbol{\tau}_{spr,R} + \mathbf{J}_{Rf,R}^T \mathbf{w}_R \\
 & \tau_i = k_{0,i} + k_{1,i} \cdot \alpha_i + k_{2,i} \cdot \alpha_i^2 + k_{3,i} \cdot \alpha_i^3 \text{ for } i = 1:6 .
 \end{aligned}$$

# Validation

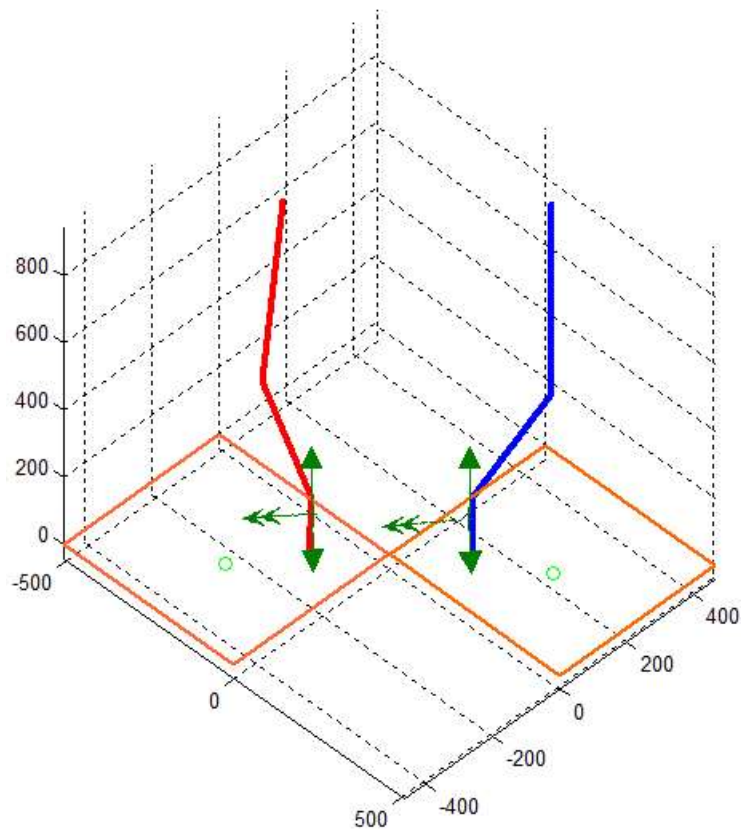
## Set-up



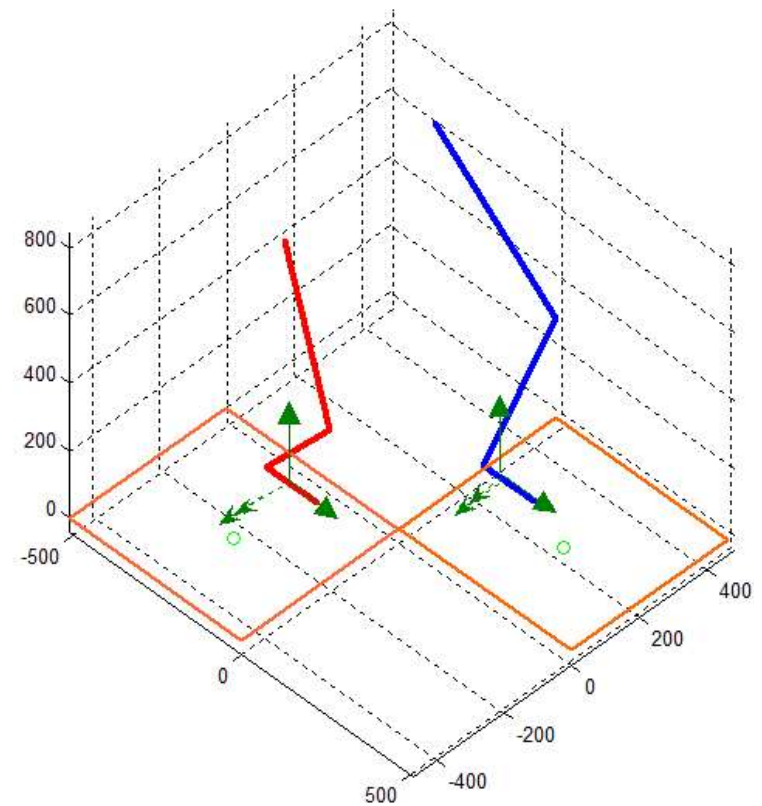


# Validation

## Procedure



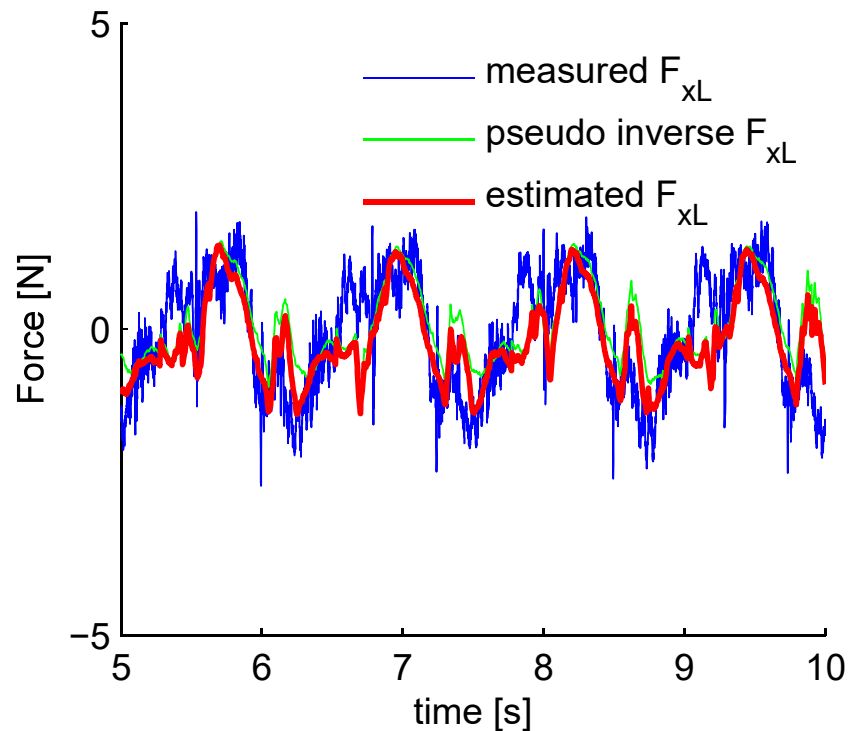
Configuration 1



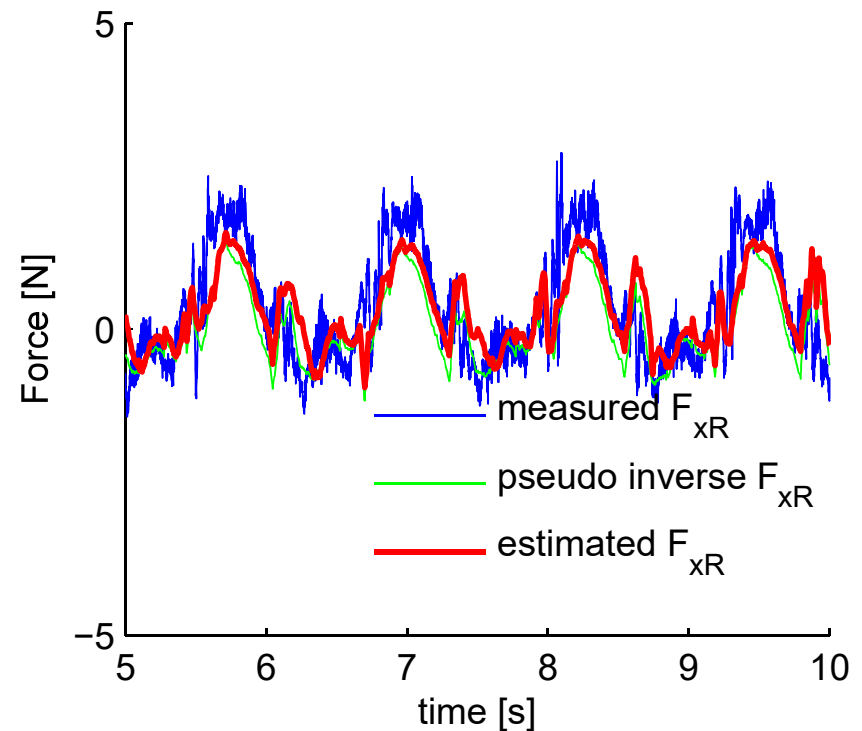
Configuration 2

# Results: Configuration 1

GRF components  $F_{xL}$  and  $F_{xR}$



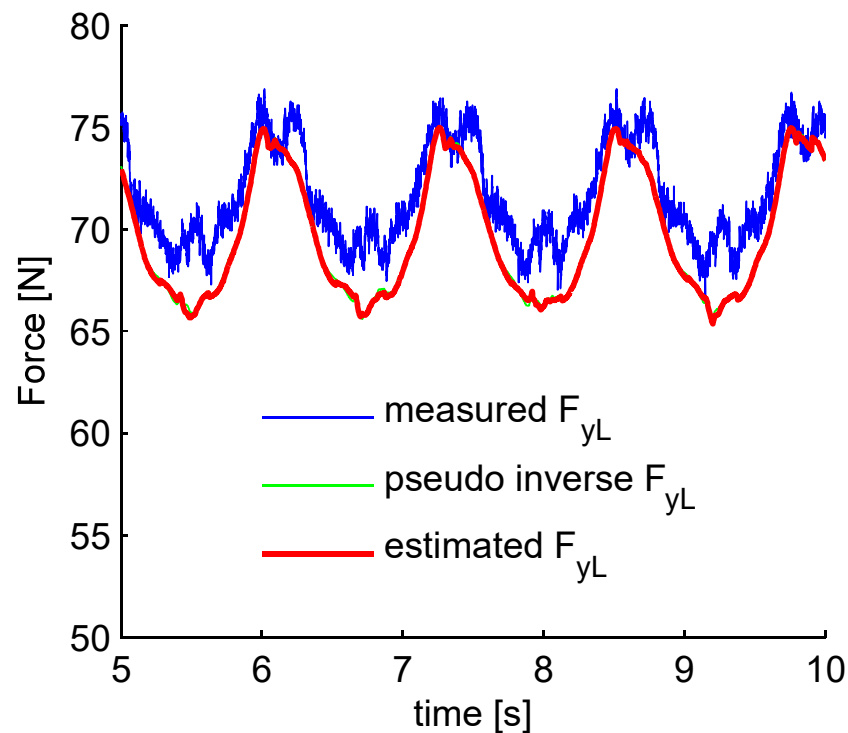
Horizontal force at left foot  $F_{xL}$



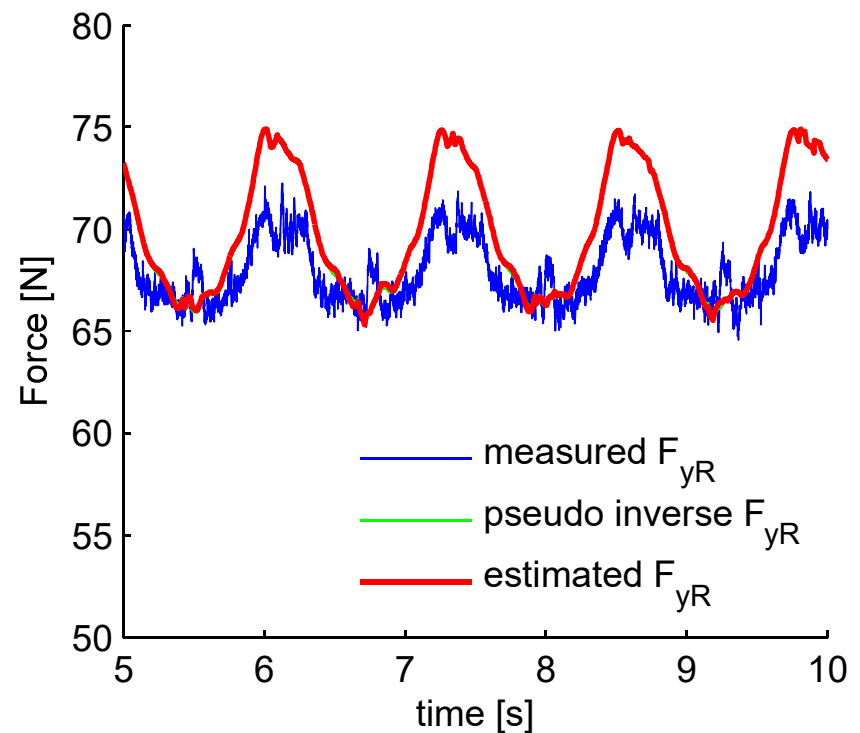
Horizontal force at right foot  $F_{xR}$

# Results: Configuration 1

GRF components  $F_{yL}$  and  $F_{yR}$



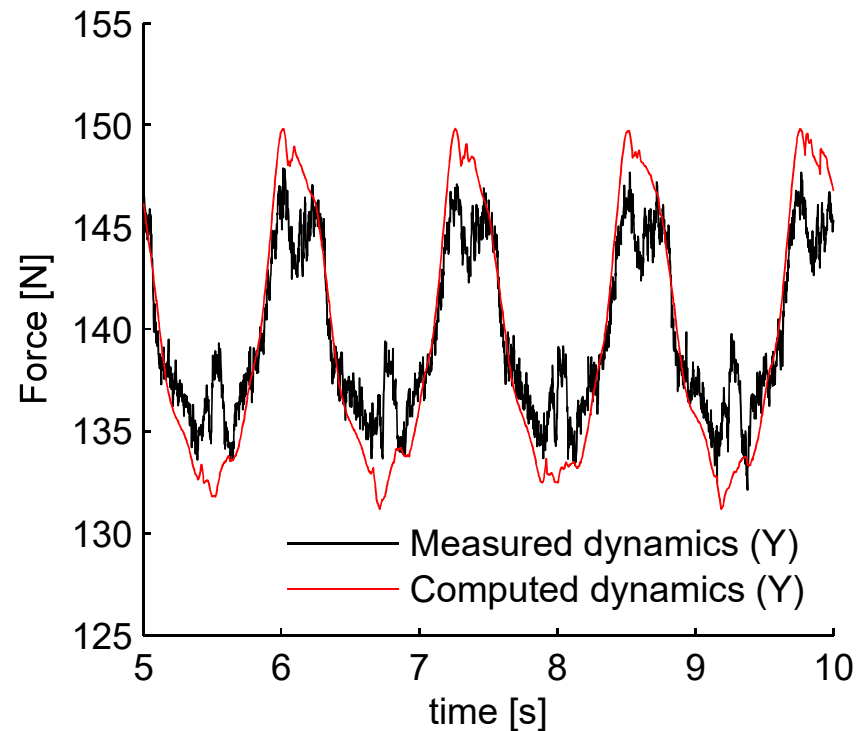
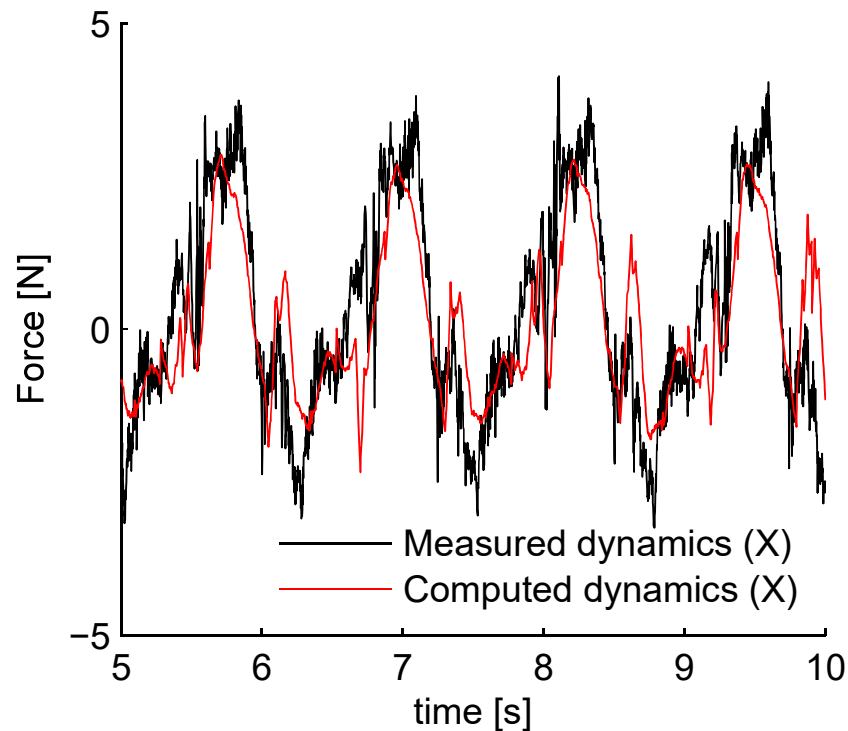
Vertical force at the left foot  $F_{yL}$



Vertical force at right foot  $F_{yR}$

# Results: Configuration 1

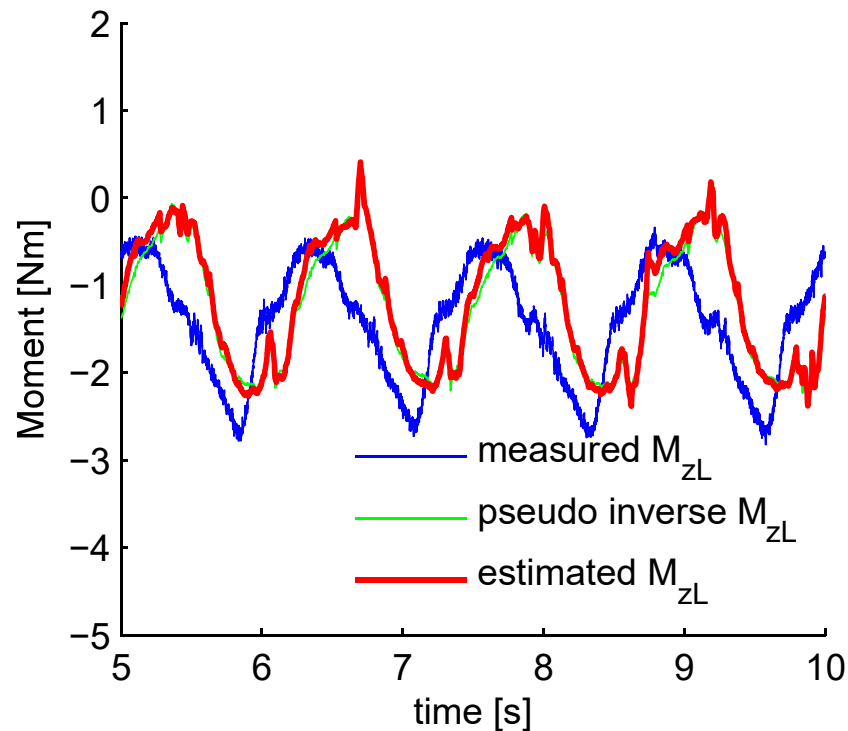
Analysis of errors in  $F_{yL}$  and  $F_{yR}$



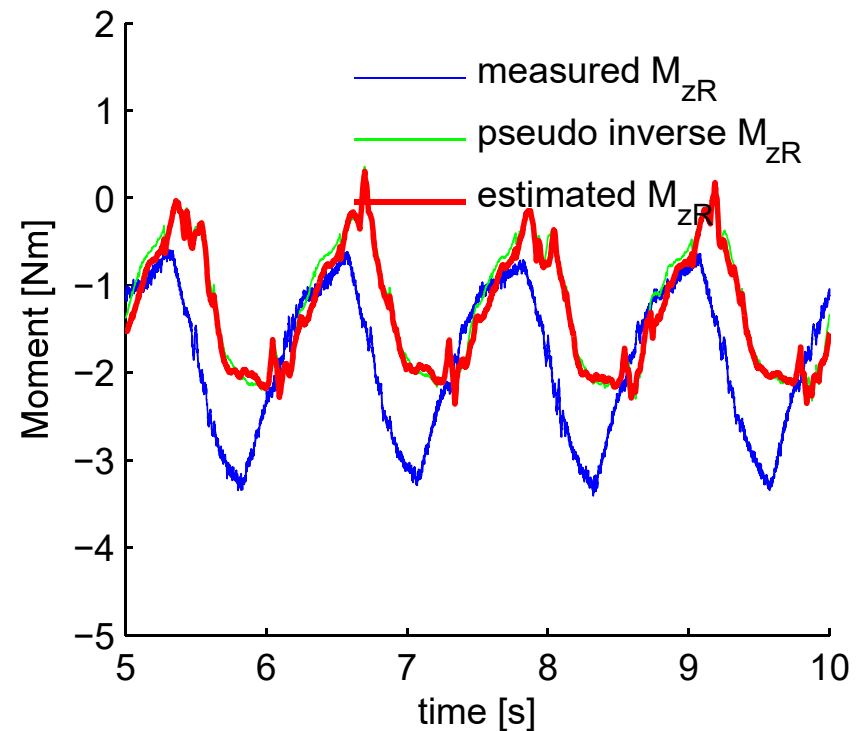
$$\blacksquare \quad \mathbf{M}_{flb} \ddot{\mathbf{q}} + \mathbf{C}_{flb} \dot{\mathbf{q}} + \mathbf{g}_{flb} = \boldsymbol{\tau}_{dyn,flb} = \mathbf{J}_{Lf,flb}^T \mathbf{w}_L + \mathbf{J}_{Rf,flb}^T \mathbf{w}_R$$

# Results: Configuration 1

GRF components  $M_{zL}$  and  $M_{zR}$



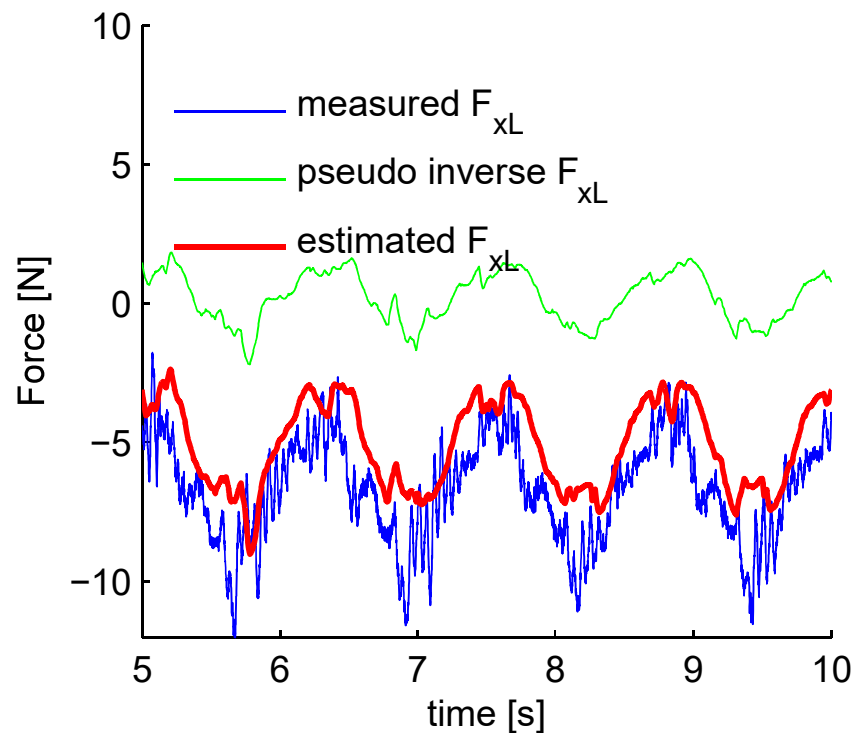
Moment about Z-axis at left foot  $M_{zL}$



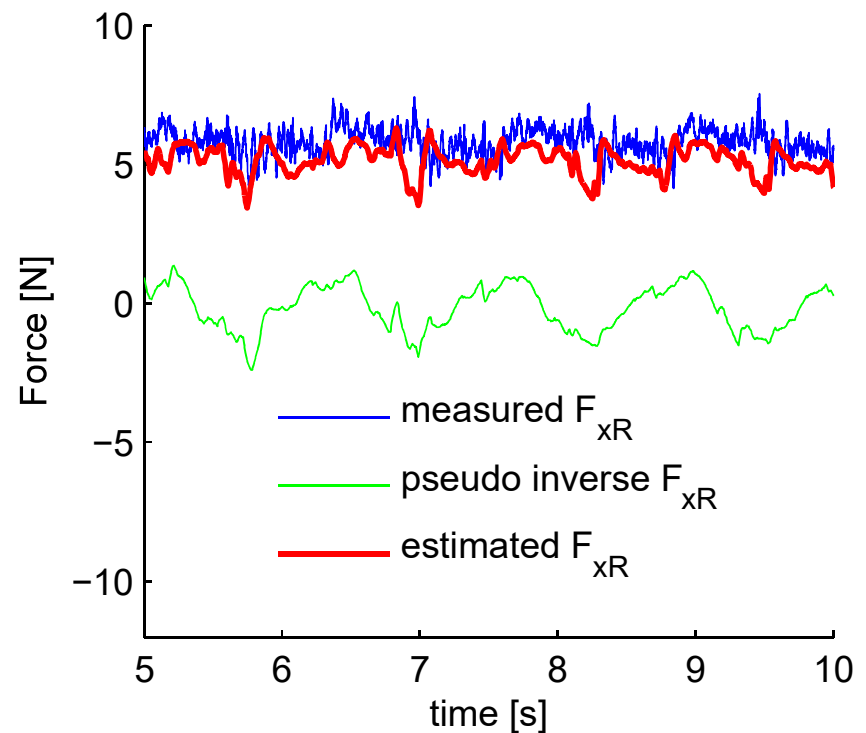
Moment about Z-axis at right foot  $M_{zR}$

# Results: Configuration 2

GRF components  $F_{xL}$  and  $F_{xR}$



Horizontal force at left foot  $F_{xL}$

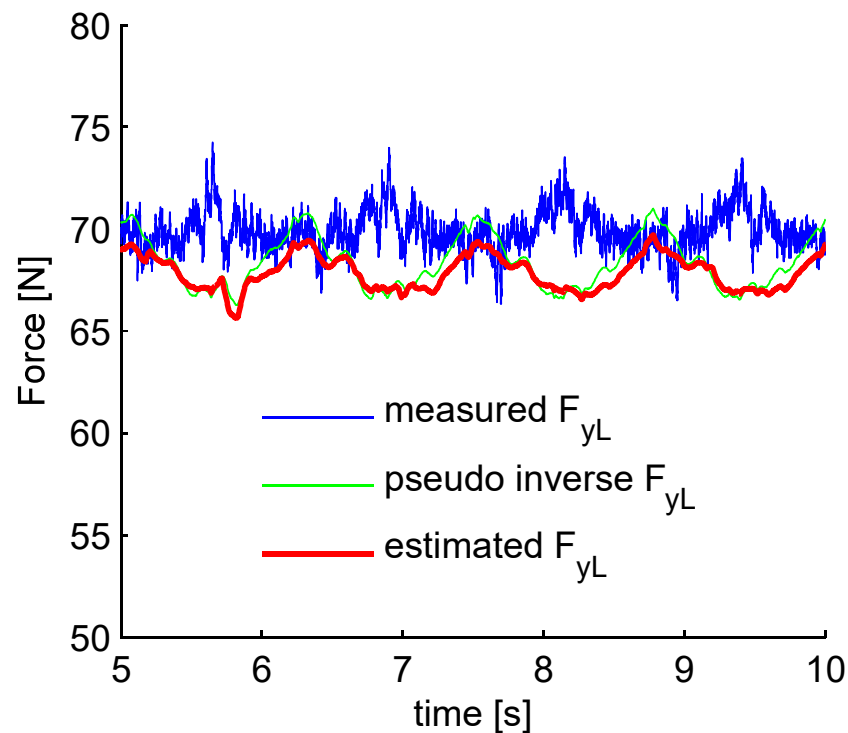


Horizontal force at right foot  $F_{xR}$

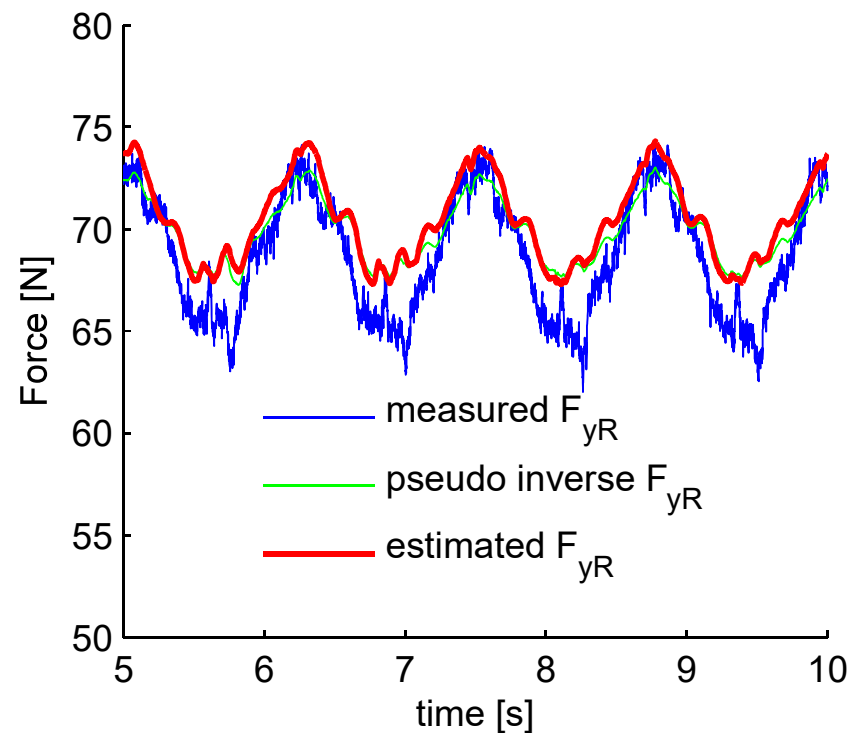


# Results: Configuration 2

GRF components  $F_{yL}$  and  $F_{yR}$



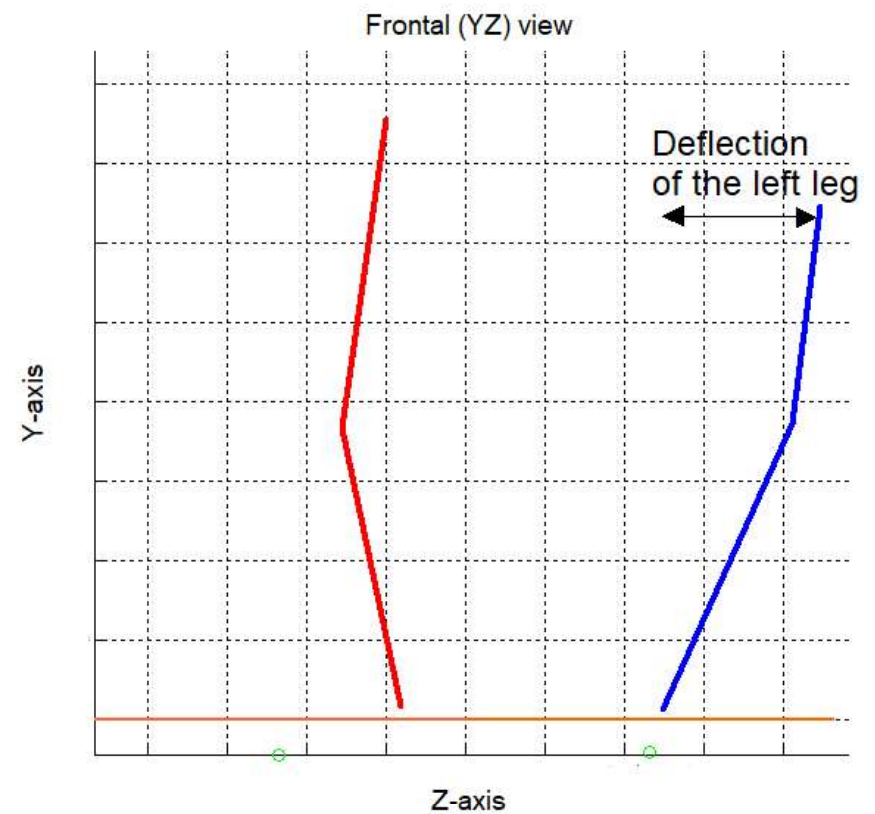
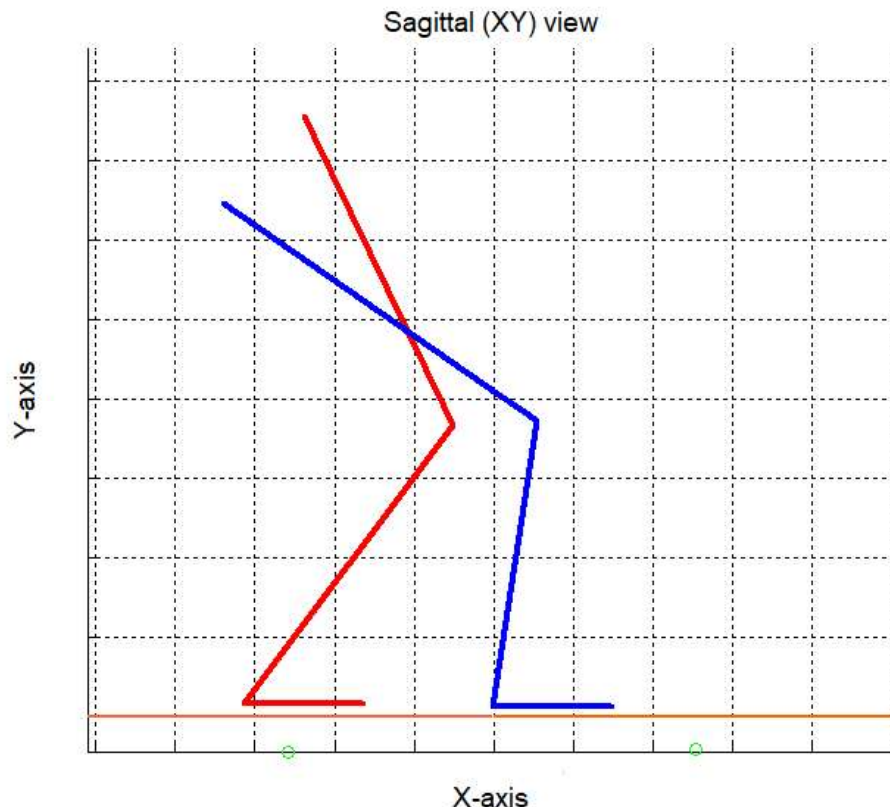
Vertical force at the left foot  $F_{yL}$



Vertical force at right foot  $F_{yR}$

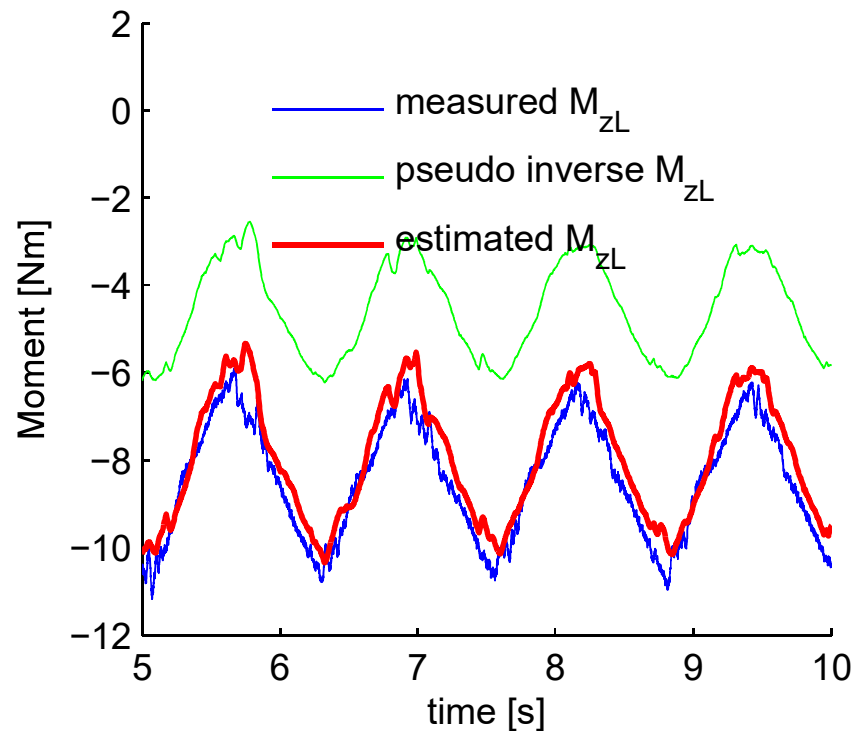
# Results: Configuration 2

Analysis of errors in  $F_{yL}$  and  $F_{yR}$

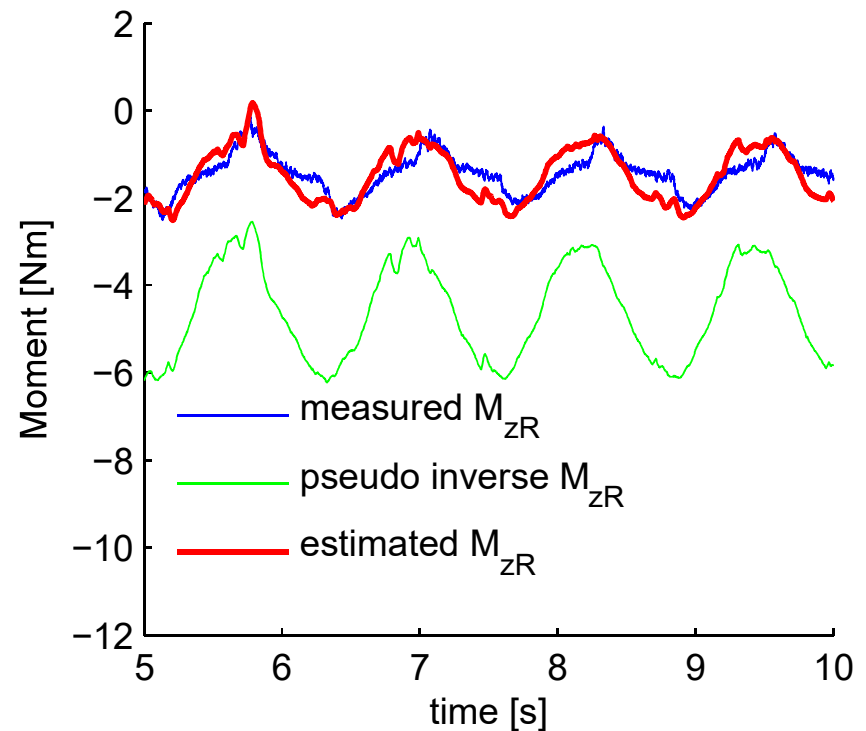


# Results: Configuration 2

GRF components  $M_{zL}$  and  $M_{zR}$



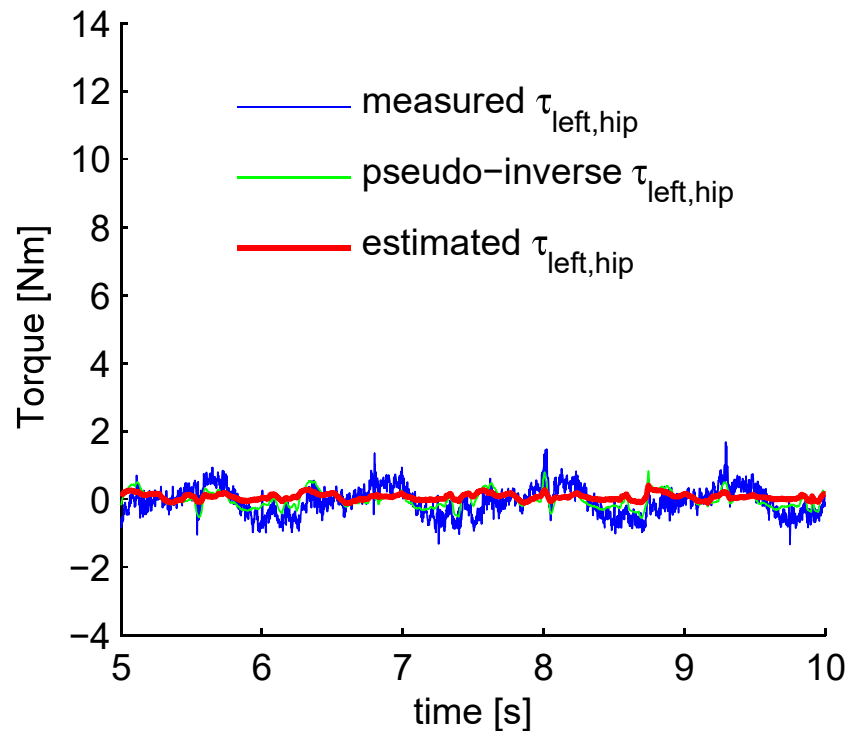
Moment about Z-axis at left foot  $M_{zL}$



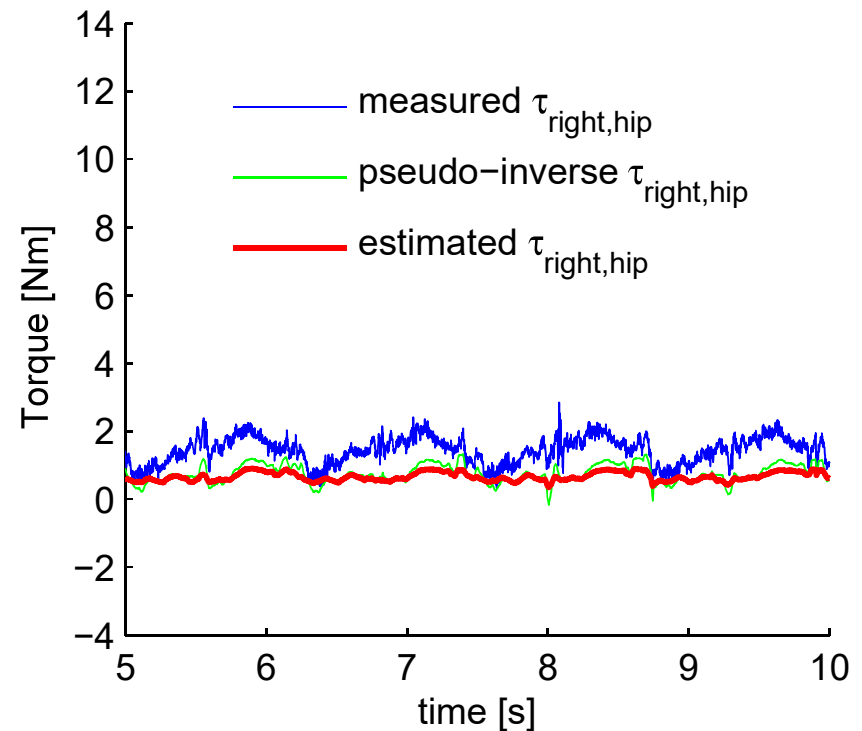
Moment about Z-axis at right foot  $M_{zR}$

# Results: Configuration 1

## Torques at the hip



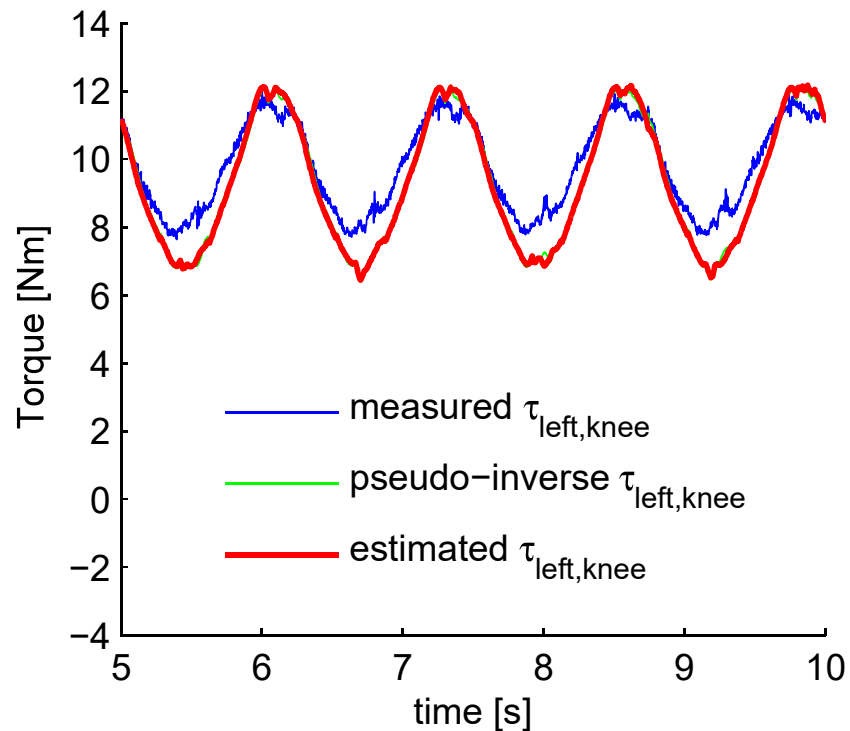
Joint torque at left hip  $\tau_{L,hip}$



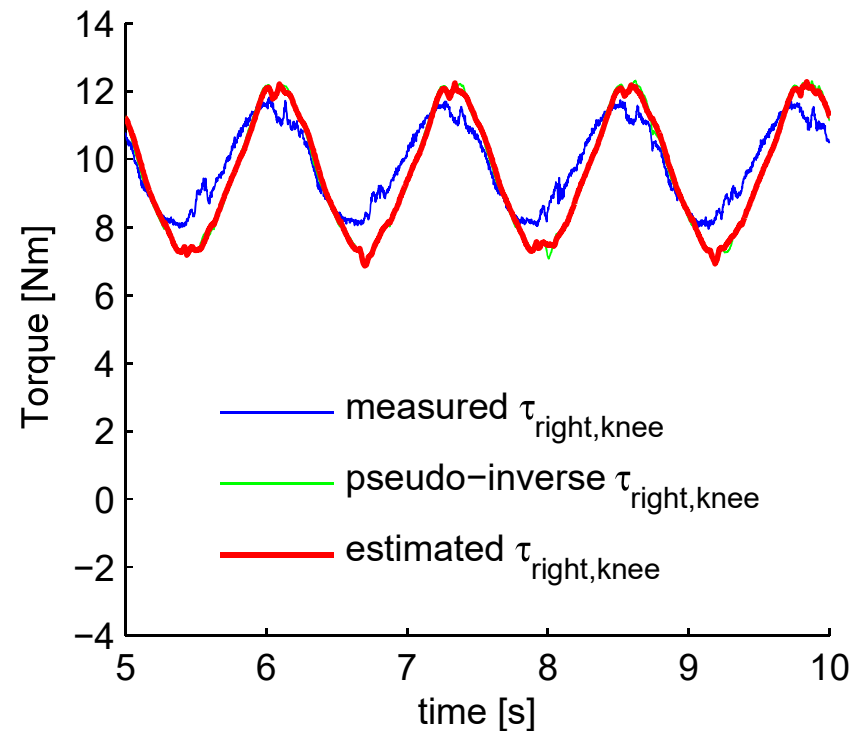
Joint torque at right hip  $\tau_{R,hip}$

# Results: Configuration 1

## Torques at the knee



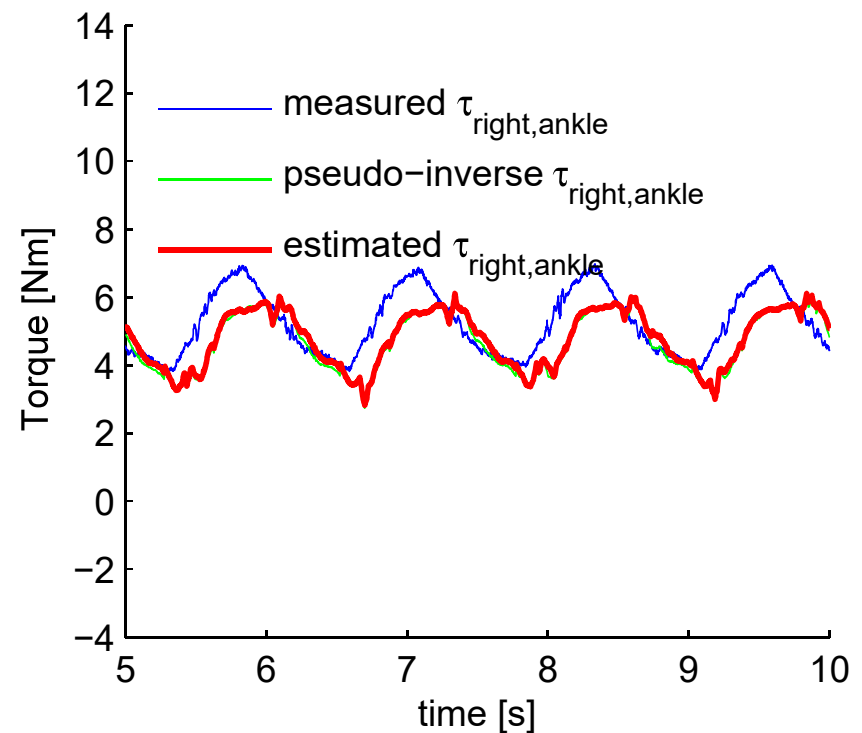
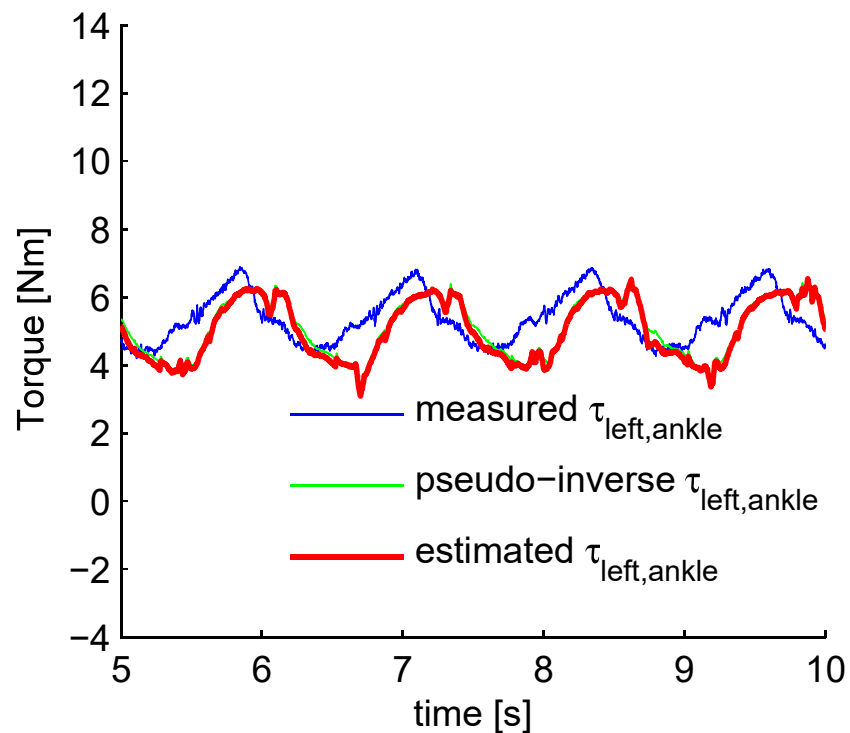
Joint torque at left knee  $\tau_{L,knee}$



Joint torque at right knee  $\tau_{R,knee}$

# Results: Configuration 1

## Torques at the ankle

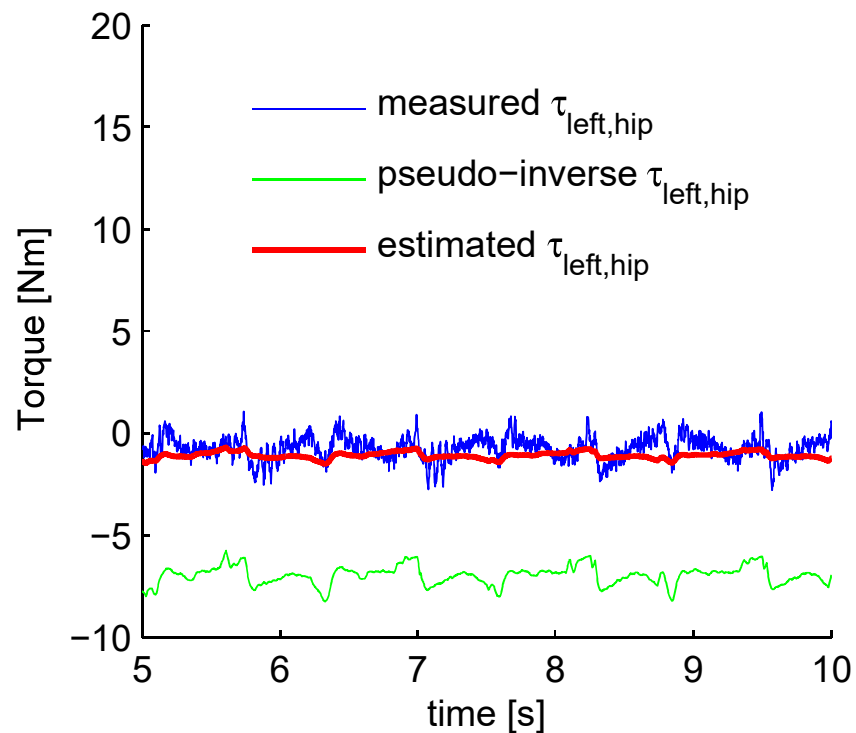


Joint torque at left ankle  $\tau_{L,ankle}$     Joint torque at right ankle  $\tau_{R,ankle}$

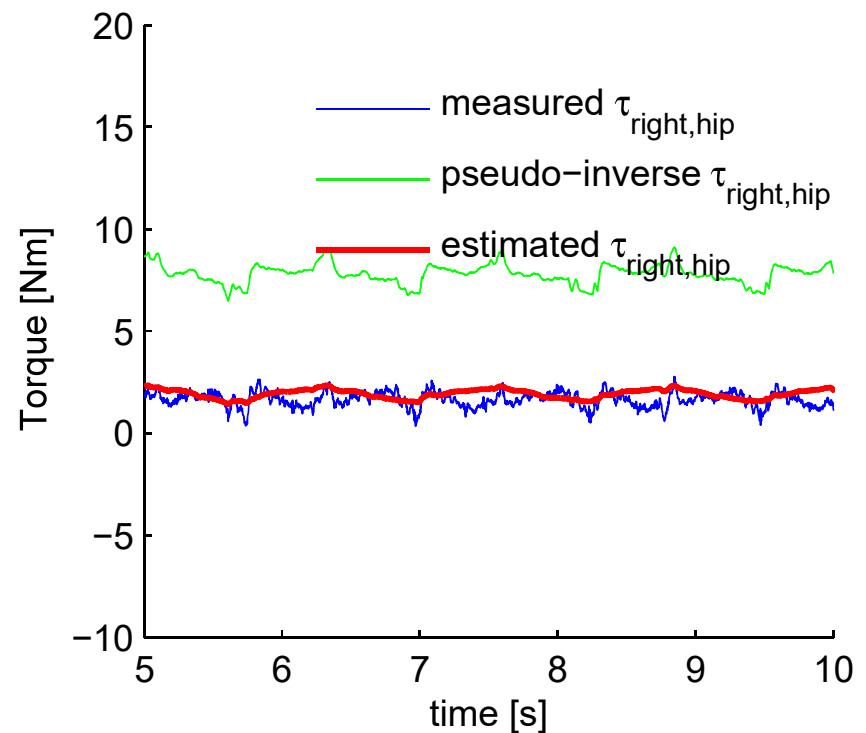


# Results: Configuration 2

## Torques at the hip



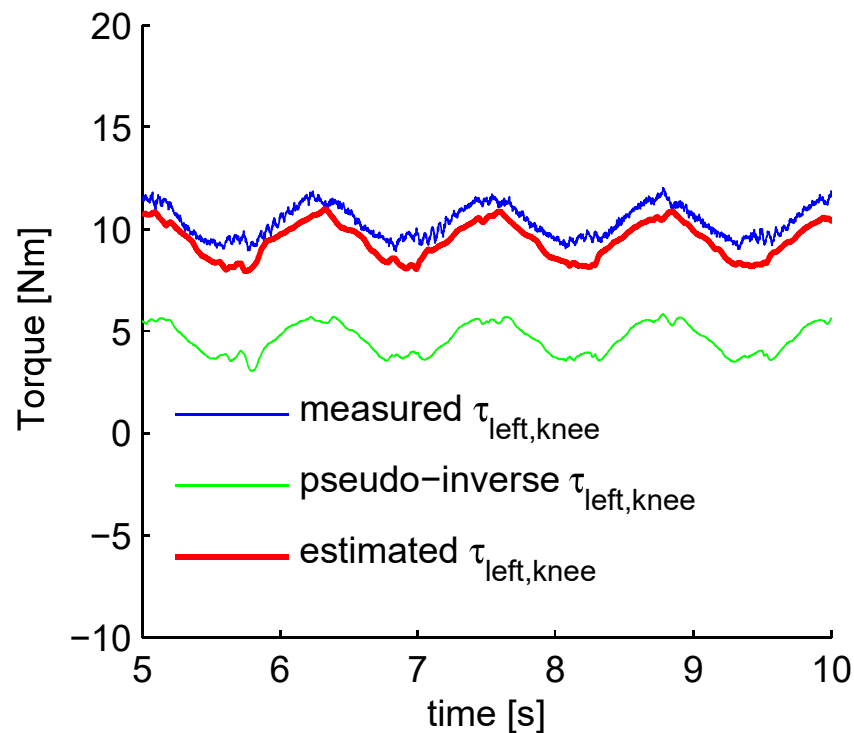
Joint torque at left hip  $\tau_{L,hip}$



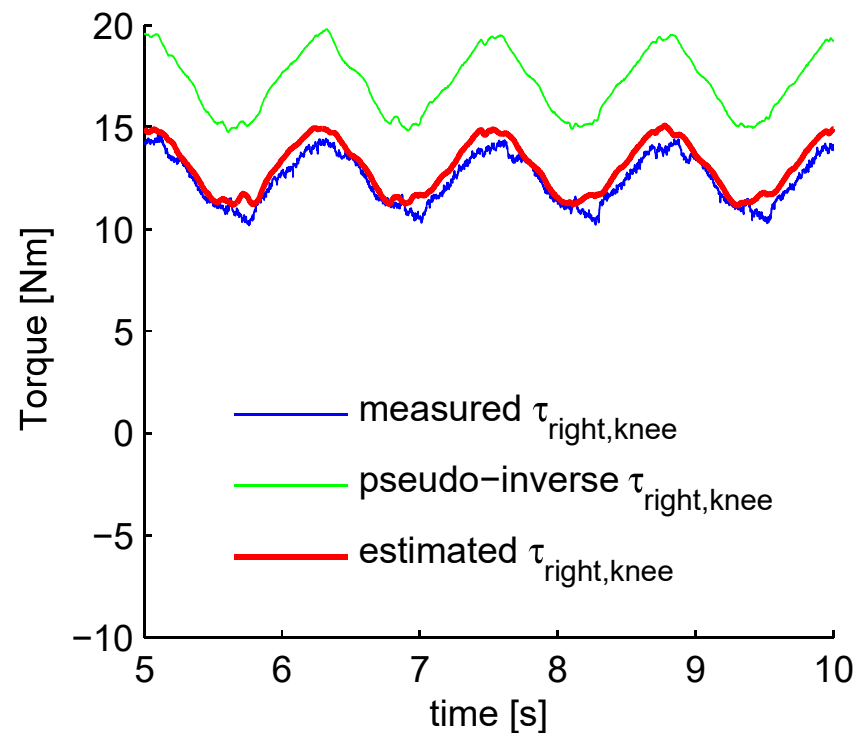
Joint torque at right hip  $\tau_{R,hip}$

# Results: Configuration 2

## Torques at the knee



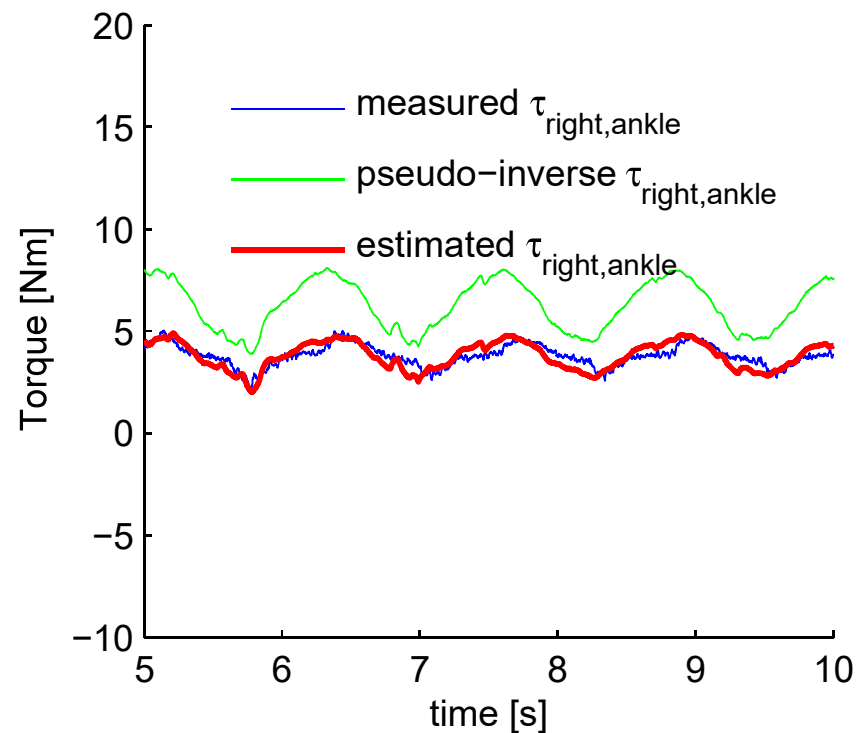
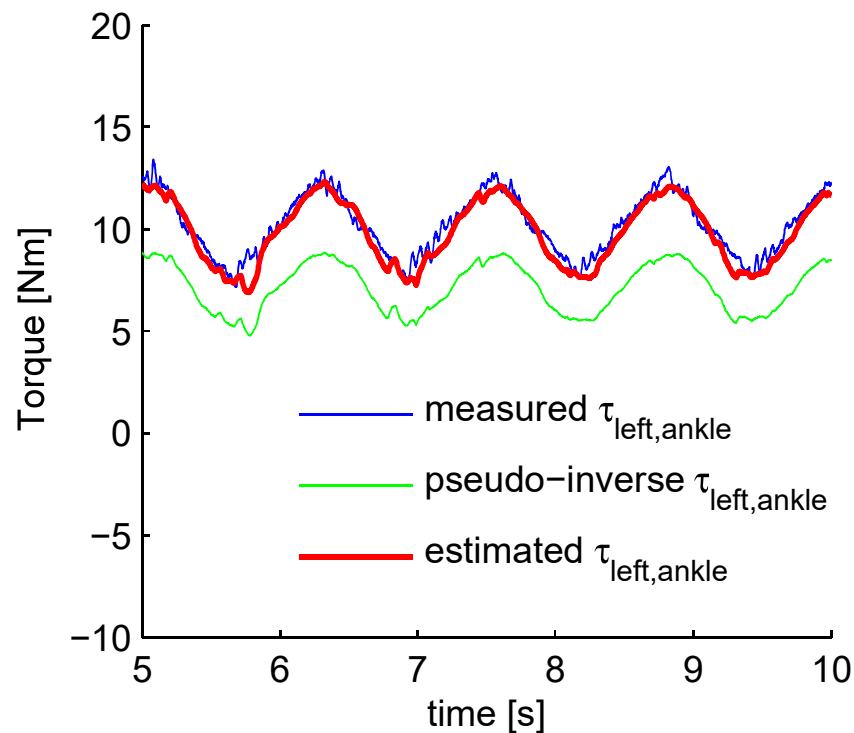
Joint torque at left knee  $\tau_{L,knee}$



Joint torque at right knee  $\tau_{R,knee}$

# Results: Configuration 2

## Torques at the ankle



Joint torque at left ankle  $\tau_{L,ankle}$     Joint torque at right ankle  $\tau_{R,ankle}$

# Conclusion

- Under-determinacy can be resolved in a **physically consistent** way by taking the stiffness model into account
- The method is quick to execute in a real-time controller
- The GRF estimation is sensitive to modelling errors
- However, the effect of modelling errors are shown to be small
- The method is shown to work under the **dynamic conditions**
- Energy stored in the system is minimized  $\Rightarrow$  **safe** for human interaction

Thank you!