Equations of Dynamics of PGS

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1 Introduction

This document contains alternative derivation(s) of equations of dynamics for the PGS given in the paper [1].

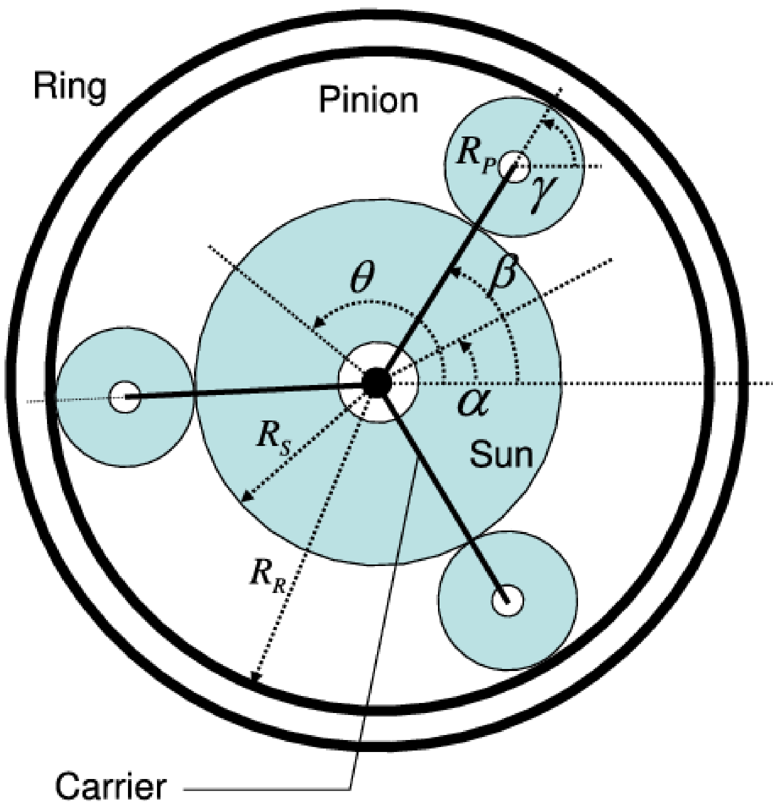

Figure 1: PGS considered in [1]. Sign convention used in this document follow from this figure (ie Anti-clockwise direction = +ve)

2 Kinematics

The equations of the kinematics are given as-

$$\dot{\beta}R_s - \dot{\gamma}R_p = \dot{\alpha}R_s \tag{1}$$

$$\dot{\beta}(R_s + 2R_p) + \dot{\gamma}R_p = \dot{\theta}R_r \tag{2}$$

The above equations can also be written as (given $R_r = R_s + 2R_p$)-

$$\beta R_s - \alpha R_s - \gamma R_p = c_1 \tag{3}$$

$$\beta(R_s + 2R_p) - \theta(R_s + 2R_p) + \gamma R_p = c_2 \tag{4}$$

where c_1 and c_2 are constants of integration.

3 Forces/Torques in the system & FBD

The external forces/torques acting on the PGS system shown in the figure 1 are-

- T_s = Applied torque on the sun
- T_c = Applied torque on the carrier
- T_r = Applied torque on the ring

The internal forces/torques acting in the PGS system are-

- F_s = Tangential action/reaction between the sun and a pinion [x3]
- F_r = Tangential action/reaction between the ring and a pinion [x3]
- F_p = Tangential action/reaction between the carrier and a pinion [x3]
- .. [similarly normal action/reaction forces, which are not detailed here]

4 Virtual Work for static case

[Give reference to virtual work formulation]
$\delta W = \delta W_E + \delta W_I = 0$

$$\delta W_E = T_s \delta\alpha + T_c \delta\beta + T_r \delta\theta \tag{5}$$

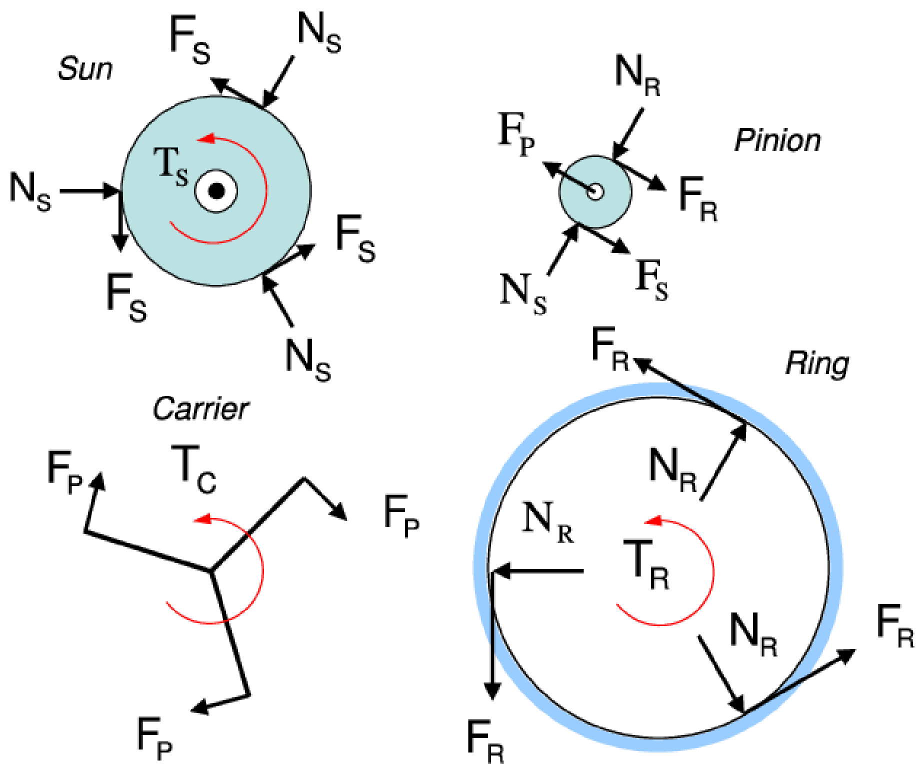

Figure 2: FBD as shown in [1]. Sign convention follows from the figure 1

$$
\begin{aligned}
\delta W_I = &+3F_s R_s \delta\alpha \\
&- 3F_s R_s \delta\beta \\
&+ 3F_s R_p \delta\gamma \\
\\
&- 3F_r R_r \delta\beta \\
&- 3F_r R_p \delta\gamma \\
&+ 3F_r R_r \delta\theta \\
\\
&+ 3F_p R_c \delta\beta \\
&- 3F_p R_c \delta\beta
\end{aligned}
\tag{6}
$$

5 Virtual Work for the dynamic case

Using the D'Alembert's principle, the acceleration terms are introduced as inertia forces.

$$\begin{aligned}
\delta W_E = &(T_s - I_s\ddot{\alpha})\delta\alpha \\
&+ (T_c - I_c\ddot{\beta})\delta\beta \\
&+ (T_r - I_r\ddot{\theta})\delta\theta \\
&- 3(M_p(R_s + R_p)^2\ddot{\beta})\delta\beta \\
&- 3(I_p\ddot{\gamma})\delta\gamma
\end{aligned} \tag{7}$$

where M_p is the mass of a pinion.

$$\begin{aligned}
\delta W_I = &+3F_s R_s\delta\alpha \\
&- 3F_s R_s\delta\beta \\
&+ 3F_s R_p\delta\gamma \\
\\
&- 3F_r R_r\delta\beta \\
&- 3F_r R_p\delta\gamma \\
&+ 3F_r R_r\delta\theta \\
\\
&+ 3F_p R_c\delta\beta \\
&- 3F_p R_c\delta\beta
\end{aligned} \tag{8}$$

6 Applying the VWM to derive equations of dynamics

By using the formula $\delta W_E + \delta W_I = 0$ and terms 7 and 8, we can derive the equations of the dynamics.

To derive the system level equations, we can rewrite certain terms like $\delta\beta$ as $\frac{\partial\beta}{\partial\alpha}\delta\alpha$ and so on. The common term $\delta\alpha$ is not equal to zero. The partial derivative terms can be replaced by their values from the equations of kinematics.

To avoid the lengthy mathematical derivation- we can also derive the equations of dynamics at the free body level (as shown in the figure 2) and then eliminate the internal forces to arrive at the equations at the system level. This derivation is given next.

Since the virtual displacements can be chosen freely- if we chose $\delta\alpha$ be non-zero virtual displacement and $\delta\beta = \delta\gamma = \delta\theta = 0$ we arrive at the equation of dynamics of the free body Sun. The process is repeated by setting one of the virtual displacement as non-zero and the remaining as zero. We therefore get 4 equations for the 4 free bodies as following -

$$(T_s - I_s\ddot{\alpha}) + 3F_sR_s = 0 \tag{9}$$

$$(T_c - I_c\ddot{\beta} - 3(M_p(R_s + R_p)^2\ddot{\beta}) - 3F_sR_s - 3F_rR_r = 0 \tag{10}$$

$$-3(I_p\ddot{\gamma}) + 3F_sR_p - 3F_rR_p = 0 \tag{11}$$

$$(T_r - I_r\ddot{\theta}) + 3F_rR_r = 0 \tag{12}$$

We have 4 equations to solve for 5 forces/torques. After eliminating F_s and F_r, we are left with 2 equations to solve for 3 applied forces/torques. We can always eliminate one of the unknowns to reduce the number of equations to one for two unknowns.

If one of any of the applied torques is measured, rest of the applied torques and internal forces can be calculated by the equations of dyanmics derived here.

References

[1] Patinya Samanuhut and Atilla Dogan. Dynamics equations of planetary gear sets for shift quality by lagrange method. In *ASME 2008 Dynamic Systems and Control Conference*, pages 353–360. American Society of Mechanical Engineers, 2008.